Finding the N N^{th} Roots of any number (real, imaginary, negative, positive, comple

Find the N N^{th} Roots of z, where:

$$z = x + i \cdot y$$

using Euler's relationship, rewrite z:

$$z = A \cdot e^{i\theta}$$

$$z^{\frac{1}{N}} = \left(A \cdot e^{i \cdot \theta}\right)^{\frac{1}{N}}$$

$$z^{\frac{1}{N}} = A^{\frac{1}{N}} \cdot e^{\frac{i \cdot \theta}{N}}$$

we recognize this a point on Euler's circle in the complex plane with magnituc (radius), $A^{1/N}$

as before, we solve for θ from the first equation defining z:

$\theta = \operatorname{atan}\left(\frac{y}{x}\right)$ again where θ can range from 0 to 2π (or $-\pi$ to π)

to find the N roots, divide the initial Euler angle (θ) by N; recall that you can (and must) add or subtract multiples of 2π to θ to find all the solutions.

Example 1. For the number i, find the three cubic roots.

Radius of circle and magnitude of all roots = $(1)^{1/3}$ = 1

$\theta = \operatorname{atan}(\infty) = \frac{\pi}{2}$ (or 90 degrees on the Euler circle)

First root is at $^{1}/3$ x θ or $\pi/6$ (i.e 30 degrees).

$R(1) = \cos\left(\frac{\pi}{6}\right) + i \cdot \sin\left(\frac{\pi}{6}\right)$ or $R(1) = \cos(30) + i \cdot \sin(30)$

if you recall your trig tables, we can fill this in as

$$R(1) = \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot i$$

For the second root, add 2π to θ:

Second root is at 1/3 x 5π/2 = 5π/6 (i.e. 150 deg)

Again recalling our trig tables,

$$R(2) = -\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot i$$

For the third rood, add another 2π to θ

Second root is at 1/3 x 9π/2 = 3π/2 (i.e. 270 deg)

$$R(3) = -i$$

We also observe that the 3 roots are equally positioned 120-degrees apart around the Euler circle. In fact this is true for all N^{th} roots. The square roots a 180 degree apart, cube roots 120 degrees, Fourth roots, 90 degrees, etc, *ad infinitum*

Example 2. With this observation, we can begin to compute some higher order roots without the aid of pencil, paper, spreadsheets or calculators. Find the three cubic roots of a positive number. Start with 1.

By inspection, one root is +1. The other 2 roots must be 120 degrees away.

$$\mathrm{Root}(2) = \frac{-1}{2} + i\frac{\sqrt{3}}{2} \qquad \mathrm{Root}(3) = \frac{-1}{2} - i\frac{\sqrt{3}}{2}$$

To find the cube root of any other real number, simply scale each solution (real a imagninary parts) by $N^{1/3}$

1. Each and every number that can be located on the complex plane (with the exception of zero) has N distinct and unique N^{th} roots.
2. Descrbing in Euler format

roots magnitude = $A^{1/N}$ where A is the complex absolute value of the original number to have roots computed.

Postion $\Phi_i = (\theta+2M\pi)/N$ where M is any integer.

Note: roots will be equally postitioned around the Euler circle

3. In (real, imaginary) format

roots = $A^{1/2}$ x $(\cos \Phi_i + i \sin \Phi_i)$

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